

# TONBRIDGE SCHOOL 

Scholarship Examination 2006

## MATHEMATICS II

Wednesday 3rd May 2006
2.00 pm

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
All the questions carry equal marks.
All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. In a recent radio interview, a Scottish gold prospector remarked that one of the more unusual properties of gold is its ability to be stretched into a long wire. Indeed, 30 grams of gold may be stretched into a long, straight, cylindrical wire from Edinburgh to Glasgow - a distance of 74 kilometres. Given that 1 cubic centimetre of gold has a mass of 19.3 grams, calculate the diameter of such a wire in millimetres.
2. (a) Find the angles $A, B, C$ of a triangle, given that they satisfy the equations:

$$
A-B=15^{\circ}, B-C=30^{\circ} .
$$

(b) Find the angles $A, B, C, D$ of a quadrilateral, given that they satisfy the equations:

$$
A-B=20^{\circ}, B-C=30^{\circ}, C-D=40^{\circ} .
$$

3. The diagram below shows a square ABCD inscribed in a circle of radius 12 cm . The two circular arcs from $A$ to $B$ are reflections of each other in the side $A B$; the same is true of the arcs on sides $\mathrm{BC}, \mathrm{CD}$ and DA. The small square PQRS is symmetrically positioned with its vertices (corners) at the midpoints of arcs. Find the area of the region labelled T enclosed by the curved lines AP and AS and the straight line PS.

4. The diagram below shows an isosceles triangle ABC with $\mathrm{AB}=\mathrm{BC}=50 \mathrm{~cm}$ and $\mathrm{AC}=60 \mathrm{~cm}$. P lies on BD , the axis of symmetry of the triangle, and $\mathrm{PD}=x \mathrm{~cm}$.
(a) If $x=15 \mathrm{~cm}$, show that $\mathrm{PA}=33.54 \mathrm{~cm}$ and $\mathrm{PB}=25 \mathrm{~cm}$.
(b) Use trial and improvement, or any other method, to find the exact value of $x$ for which $\mathrm{PA}=\mathrm{PC}=\mathrm{PB}$.

5. In the diagram below, triangle ABC has a right-angle at C and triangles $\mathrm{BPQ}, \mathrm{APR}, \mathrm{PQR}$ are isosceles with $\mathrm{BP}=\mathrm{BQ}, \mathrm{AP}=\mathrm{AR}$ and $\mathrm{RP}=\mathrm{RQ}$ as shown.
(a) If $\mathrm{a}=10 \propto$, show that PQR is a right-angled triangle.
(b) Show that, whatever the value of angle $\mathrm{a}, \mathrm{PQR}$ is a right-angled triangle.

6. The figure below depicts a large outer rectangle subdivided into 9 square pieces which fit together as shown without gaps. If the smallest square (labelled S) has side-length 1 cm , what is the side-length of the largest square (labelled L )?

7. Study carefully the table below in which the entries in columns $\mathrm{A}, \mathrm{B}, \mathrm{C}$ are related to those in columns D, E, F which, in turn, are related to those in columns G, H, I.

|  | $\boldsymbol{A}$ | $\boldsymbol{B}$ | $\boldsymbol{C}$ | $\boldsymbol{D}$ | $\boldsymbol{E}$ | $\boldsymbol{F}$ | $\boldsymbol{G}$ | $\boldsymbol{H}$ | $\boldsymbol{I}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{3}$ | 4 | 9 | 16 | 2 | 10 | 14 | 4 | 100 | 196 |
| $\boldsymbol{4}$ | 9 | 16 | 25 | 7 | 17 | 23 | 49 | 289 | 529 |
| $\mathbf{5}$ | 16 | 25 | 36 | 14 | 26 | 34 | 196 | 676 | 1156 |
| $\boldsymbol{6}$ | - | - | - | - | - | - | - | - | - |
| $\mathbf{7}$ | - | - | - | - | - | - | - | - | - |
| $\boldsymbol{n}$ |  |  |  |  |  |  |  |  |  |

(a) What are the entries (shown as dashes in the table) corresponding to the entries 6 and 7 in the left-hand column?
(b) If $n$ is the entry in the left-hand column, find the entries in columns A, B, C, D, E, F, $\mathrm{G}, \mathrm{H}, \mathrm{I}$ in terms of $n$.
(c) If $\mathrm{H}=84100$, what are the entries in columns $G$ and $I$ ?
(d) If $\mathrm{I}=329476$, what are the entries in columns $G$ and $H$ ?
(e) What is the connection between the numbers in columns G, H, and I?

